Hot X: Algebra Exposed

Solution Guide for Chapter 19

Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

DTM from p.279

2. Looking at just the first term, we know that the cube root of –1 equals **–1**, and that the cube root of 1 equals **1**. And if you didn't know that the cube root of 125 equals **5**, then just try doing the prime factorization, and you'd get: 125 = 5 X 5 X 5. Tada!
So our problem becomes: –1 + 1 + 5 which of course equals **5**.
Answer: 5

3. The square root of 81 is **9**, and the 4^{th} root of 81 is just **3**. What's the square root of 0.0081? Well, you can figure it's something like 0.09 or 0.9 – so let's test them! (Remember counting decimal places for decimal multiplication? If not, check out chapter 10 in "Math Doesn't Suck.")
0.9 X 0.9 = (two decimal places) = 0.81 Nope!
0.09 X 0.09 = (four decimal places) = 0.0081 Bingo!

So the square root of 0.0081 is **0.09**, and we get:
9 – 3 + 0.09 = 6.09
Answer: 6.09

4. The first term of this one should stop us in our tracks. We can't have a square root of a negative number! So this problem does not have a real number answer.

Answer: Not a real number

5. The cube root of –64 is **–4**, the cube root of –27 is **–3**, and what's the cube root of 0.001? Hm, maybe **0.1**? Let's see: 0.1 X 0.1 X 0.1 = (three decimal places) = 0.001.
Yep! Now, watch those negative signs as we stick this stuff back in:

$-4 - (-3) + 0.1 \quad = \quad -4 + 3 + 0.1 \quad = \quad -1 + 0.1$

Hm, a tricky last step. Just imagine the opposite problem: 1 – 0.1 = 0.9. And now it's a bit easier to see how –1 + 0.1 = **–0.9**

Answer: –0.9

DTM from p.284-285

2. We know that 50 = 25 X 2, so it's easy to see how we can factor out the perfect square 25; we get: $\sqrt{50} \;=\; \sqrt{25\cdot 2} \;=\; \sqrt{25}\cdot\sqrt{2} \;=\; \mathbf{5\sqrt{2}}$

Answer: $\mathbf{5\sqrt{2}}$

3. Is there a perfect square factor in 84? Let's factor it. We know from our times tables that 84 = 12 X 7. Oh, and 12 has a factor of 4 in it, which is a perfect square.
In fact: 84 = 3 X 4 X 7. And we can see that 4 is the only perfect square factor, so it's the only thing we can pull out: $\sqrt{84} \;=\; \sqrt{4\cdot 3\cdot 7} \;=\; 2\sqrt{3\cdot 7} \;=\; \mathbf{2\sqrt{21}}$

Answer: $\mathbf{2\sqrt{21}}$

4. Dealing with just the first term, we can do the factorization and get:
108 = 12 X 9 = 3 X 4 X 9. So there are two perfect squares, 4 and 9. And so we get:

$\sqrt{108} \;=\; \sqrt{4\cdot 9\cdot 3} \;=\; \sqrt{4}\cdot\sqrt{9}\cdot\sqrt{3} \;=\; 2\cdot 3\cdot\sqrt{3} \;=\; \mathbf{6\sqrt{3}}$

Now for the second term, we start by factoring 225. How do we start? Well, it's certainly divisible by 5. Dividing it by 5, we see that: 225 = 5 X 45.

Oh, so in other words: 225 = 5 X 5 X 9, or rewritten: 225 = 25 X 9. Hey, it's a product of only perfect squares? That actually means 225 itself is a perfect square; check it out: $\sqrt{225} = \sqrt{25 \cdot 9} = \sqrt{25} \cdot \sqrt{9} = 5 \cdot 3 =$ **15**. And in fact, 15 X 15 = 225.

Putting both halves of our answer together, we get: $\mathbf{6\sqrt{3} + 15}$.

Answer: $\mathbf{6\sqrt{3} + 15}$

5. Let's start by factoring 324. Using divisibility tricks like the ones on p.9 of Math Doesn't Suck (and also found in the "extras" page on my websites), we can tell that 324 is divisible by 9, since 3 + 2 + 4 = 9 (this trick only works for 3 and 9!). Let's divide 9 into 324, and we get: 324 = 9 X 36. Both are perfect squares! So we've discovered another perfect square, 324! We get: $\sqrt{324} = \sqrt{9 \cdot 36} = \sqrt{9} \cdot \sqrt{36} = 3 \cdot 6 =$ **18**.

And yep! 18 X 18 = 324.

Next half of the problem; we need to factor 68. Divisibility tricks tell us that this is divisible by 4, and dividing it in, we get: 68 = 4 X 17. So:

$\sqrt{68} = \sqrt{4 \cdot 17} = \sqrt{4} \cdot \sqrt{17} = \mathbf{2\sqrt{17}}$

Combining both halves of our answer, we get: $\mathbf{18 + 2\sqrt{17}}$

Answer: $\mathbf{18 + 2\sqrt{17}}$

DTM from p.287

2. Let's multiply these, and see what happens: $\sqrt{18} \cdot \sqrt{2} = \sqrt{18 \cdot 2} = \sqrt{36} =$ **6**. Looks pretty simplified to me!

Answer: 6

3. Collecting coefficients to the front, we get: $2 \cdot 4 \cdot \sqrt{30} \cdot \sqrt{5}$, and now we multiply the coefficients and radical parts separately: $8 \cdot \sqrt{30 \cdot 5}$

But instead of multiplying the 30 and 5 together, let's rewrite 30 as 6 X 5, remembering that we're hunting for perfect squares! Look what happens:

$8 \cdot \sqrt{30 \cdot 5} \quad = \quad 8 \cdot \sqrt{6 \cdot 5 \cdot 5} \quad = \quad 8 \cdot \sqrt{25} \cdot \sqrt{6} \quad = \quad 8 \cdot 5 \cdot \sqrt{6} \; = \; \mathbf{40\sqrt{6}}$

Nice!

Answer: $\mathbf{40\sqrt{6}}$

4. Again, we'll collect and multiply the coefficients separately from the radical parts:

$2 \cdot 5 \cdot \sqrt{21} \cdot \sqrt{7} \; = \; 10\sqrt{21 \cdot 7}$

Taking the hint, we'll write 21 as 3 X 7, and we get:

$10\sqrt{21 \cdot 7} \; = \; 10\sqrt{3 \cdot 7 \cdot 7} \quad = \quad 10\sqrt{49} \cdot \sqrt{3} \; = \; 10 \cdot 7 \cdot \sqrt{3} \; = \; \mathbf{70\sqrt{3}}$ Done!

Answer: $\mathbf{70\sqrt{3}}$

DTM from p.290

2. Writing the radical part as just x, then since we know that $4x + 3x = 7x$, that means we also know that: $4\sqrt{5} + 3\sqrt{5} = \mathbf{7\sqrt{5}}$. It's just a little trick I like to use to make these square root expressions easier to deal with!

Answer: $\mathbf{7\sqrt{5}}$

3. The radical parts don't match so we can't add these as they are. Let's see what happens when we simplify the first term:

$2\sqrt{20} \quad = \quad 2\sqrt{4 \cdot 5} \quad = \quad 2 \cdot \sqrt{4} \cdot \sqrt{5} \quad = \quad 2 \cdot 2 \cdot \sqrt{5} \quad = \quad \mathbf{4\sqrt{5}}$

Now let's simplify the second term: $\sqrt{45} \quad = \quad \sqrt{9 \cdot 5} \quad = \quad \sqrt{9} \cdot \sqrt{5} \quad = \quad \mathbf{3\sqrt{5}}$

So our problem has become: $4\sqrt{5} + 3\sqrt{5}$

Hey, that's the same problem as #2!

Answer: $\mathbf{7\sqrt{5}}$

4. Since we learned in #3 that $2\sqrt{20} = 4\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}$, we know *this* problem simplifies like this: $2\sqrt{20} - \sqrt{45} \quad = \quad 4\sqrt{5} - 3\sqrt{5}$. Make sense? Now that the radical parts are the same, we can subtract just like we would do in the problem $4x - 3x = x$, and we get: $4\sqrt{5} - 3\sqrt{5} \; = \; \boldsymbol{\sqrt{5}}$

Answer: $\boldsymbol{\sqrt{5}}$

5. Again, since we learned in #3 that $2\sqrt{20} = 4\sqrt{5}$ and $\sqrt{45} = 3\sqrt{5}$, we know *this* problem simplifies like this: $\sqrt{45} - 2\sqrt{20} = 3\sqrt{5} - 4\sqrt{5}$. And how do we do this subtraction? When in doubt, rewrite the similar radicals variables to help it not look so foreign! We know that $3x - 4x = -x$, right? So that means: $3\sqrt{5} - 4\sqrt{5} = \boldsymbol{-\sqrt{5}}$

Answer: $\boldsymbol{-\sqrt{5}}$

6. We must obey the order of operations! So we shouldn't be tempted to do that subtraction before we do the multiplication. Doing the multiplication $3\sqrt{5} \cdot \sqrt{4}$, we get:

$$3\sqrt{5} \cdot \sqrt{4} \quad = \quad 3\sqrt{5} \cdot 2 \quad = \quad 3 \cdot 2 \cdot \sqrt{5} \quad = \quad 6\sqrt{5}$$

So our entire problem has become: $4\sqrt{5} - 6\sqrt{5}$. Great! The radical parts are the same, so we can subtract them now.

And since we know that $4x - 6x = -2x$, we also know that: $4\sqrt{5} - 6\sqrt{5} = \boldsymbol{-2\sqrt{5}}$. Done!

Answer: $\boldsymbol{-2\sqrt{5}}$