Hot X: Algebra Exposed

Solution Guide for Chapter 2

Here are solutions for the “Doing the Math” exercises in *Hot X: Algebra Exposed*!

DTM from p.20-21

2. Dealing with the coefficients first, the GCF of 4 & (the invisible) 1 is just **1**. Let’s look at the variables one at a time. Comparing a^2 and a, $\boldsymbol{a}$ has the lower exponent, so it “wins”! Comparing b and b^2, $\boldsymbol{b}$ wins! So the GCF of the two expressions is $1 \cdot a \cdot b = ab$. Answer: $\boldsymbol{ab}$.

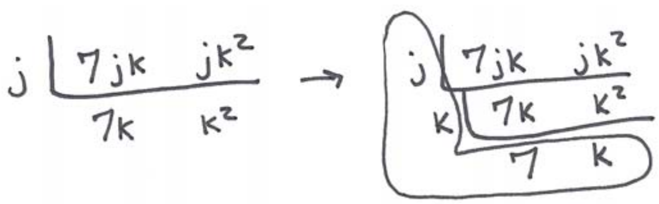
3. The GCF of 14 & 28 is **14**. Then comparing x^3 and x, $\boldsymbol{x}$ wins! Comparing y^2 with y^2, $\boldsymbol{y^2}$ “wins”! Those are the only variable factors that are shared by *both* expressions. So the GCF is $14 \cdot x \cdot y = \mathbf{14}\boldsymbol{xy^2}$. Answer: $\mathbf{14}\boldsymbol{xy^2}$.

4. The GCF of 5 and 15 is 5. Since our original two expressions share no variable factors, we’re done! The GCF of the two original expressions is 5. Answer: **5**

5. The GCF of 6 and 4 is **2**. Comparing a^2 and a^2, $\boldsymbol{a^2}$ “wins”! Comparing b and b, $\boldsymbol{b}$ “wins!” So the entire GCF is $2 \cdot a^2 \cdot b = \mathbf{2}\boldsymbol{a^2b}$. Answer: $\mathbf{2}\boldsymbol{a^2b}$

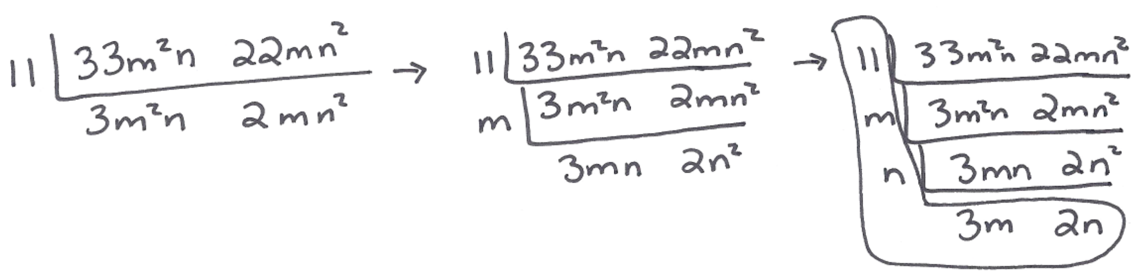
DTM from p.26

2. Drawing our first layer, let's notice the common factor of "*j*", write it on the side, and then we are left with $7k$ and k^2 to write underneath in a new layer. Then let's pull out the comment factor of "*k*" and our final layer is 7 and *k*, which have no common factors.



The GCF of the original two expressions is found by multiplying the stuff on the left, which is: $j \cdot k = jk$. And the LCM of the original two expressions can be found by multiplying everything in the big "L" shape – the stuff on the left and the stuff on the bottom layer: $j \cdot k \cdot 7 \cdot k = \mathbf{7jk^2}$. Answer: **The GCF is *jk* and the LCM is $7jk^2$.**

3. Let's draw a "shelf" under the two original expressions, and what common factor do we notice first? How about 11? So we'll write that on the side, and we are left with $3m^2n$ and $2mn^2$ as the next layer. These two new expressions share a factor of "*m*" so let's write that on the side and our newest layer becomes $3mn$ and $2n^2$. Do these two expressions still share a common factor? Yep, *n*! So let's pull that out, and the final layer ends up being $3m$ and $2n$.



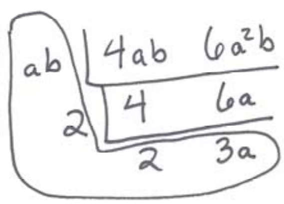
The GCF will be the product of the stuff on the left: $11 \cdot m \cdot n = \mathbf{11mn}$, and the LCM will be the product of everything in the big "L" shape: $11 \cdot m \cdot n \cdot 3m \cdot 2n = \mathbf{66m^2n^2}$.
Answer: **The GCF is 11*mn* and the LCM is $66m^2n^2$.**

4. First let's find the GCF of these three terms, which we can use the cake method on (because it <u>is</u> okay to use the cake method to find the GCF of three terms at once – just

not for the LCM!), but it's actually pretty fast to eyeball this one. The GCF of 4, 6, and 9 is just **1**; they share no common factors. The only variable factor they have in common is *b*. Comparing *b*, *b*, and *b*, ***b*** wins! So the GCF of all three terms is $1 \cdot b = \boldsymbol{b}$.

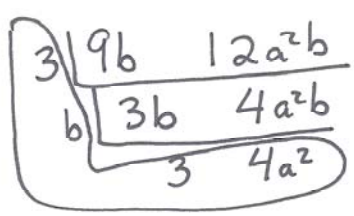
Next, we need to find the LCM of all three original terms. But we know that we can't use the cake method to find the LCM of three terms at once; most of the time it would give the wrong answer! So we first find the LCM of two of the terms, and then find the LCM of that answer and the third, unused original term. (See p.25 to review how this works.)

So let's take $4ab$ and $6a^2b$, and find their LCM. We could start this a few ways, but just to see how it works, let's start by pulling out the common factor *ab*. Then we'll be left with 4 and 6*a* left over to write on the next layer. They still have a common factor of 2, so let's pull that out, and we're left with 2 and 3*a* as the final layer, which has no common factors left.



The GCF is the product of the big "L", which is: $ab \cdot 2 \cdot 2 \cdot 3a = \mathbf{12}\boldsymbol{a^2b}$.
Next, we use this answer and the third original term, 9*b*, and find their LCM. That answer will indeed be the LCM of all three original terms.

Drawing a whole new cake to find the LCM of $9b$ and $12a^2b$, we can start by pulling out 3, which is a common factor, and we're left with $3b$ and $4a^2b$ as the next layer. They still share a common factor of b, so let's pull that out, and our final layer will be 4a and 3.



The LCM is the product of the big "L" shape, which is: $3 \cdot b \cdot 3 \cdot 4a^2 = \mathbf{36}\boldsymbol{a^2b}$. And this is indeed the LCM of all three original terms!
Answer: **The GCF is *b* and the LCM is 36$\boldsymbol{a^2b}$.**