Hot X: Algebra Exposed

Solution Guide for Chapter 20

Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

DTM from p.299

2. Count 'em, we currently have 3 terms. Combining like terms, we get: $6wz^2 - 1$. Its degree is 3, since *w*'s exponent is 1 and 1 + 2 = **3**. Also, it now has two terms, so it's a binomial.

Answer:

Part a: three terms

Part b: $6wz^2 - 1$

Part c: Degree = 3; it has two terms; binomial

3. It currently has four terms, but when we combine like terms, the $3x$'s cancel each other away! And we get: 9 – 1 = 8. Constants have a degree of zero, and this "polynomial" is just a monomial now, since it only has one term left.

Answer:

Part a: four terms

Part b: 8

Part c: Degree 0; it has one term; monomial

4. We currently have 5 terms. Combining like terms, we get: $e^4 - 2d^2e^3 + 6$, and putting it in order of degree, that's $-2d^2e^3 + e^4 + 6$. Its degree is 2 + 3 = **5**. And now, since it has three terms, it's a trinomial.

Answer:

Part a: five terms

Part b: $-2d^2e^3 + e^4 + 6$

Part c: degree 5; three terms; trinomial

DTM from p.301

2. So, let's set up our birthday cake layer, and just focus on factoring one factor from each term to start. Seems all the terms have some *a*'s in them, and in fact, the lowest power of the *a*'s is 4, so we know we can factor out a^4 from each term. Next, in our second layer, you can see that we have *b*'s left in every term, and again, the lowest power on the *b*'s is 4, so we know we can pull out b^4 from each term.

$$\begin{array}{r|l} a^4 & 2a^5b^4 + 6a^4b^5 + 9b^4a^5c \\ \hline b^4 & 2ab^4 + 6b^5 + 9b^4ac \\ \hline & 2a + 6b + 9ac \end{array}$$

What we're left with, $\mathbf{2a + 6b + 9ac}$, has no factors common to all the terms, so that's what will go in the parentheses! And of course, the stuff we factored out, $\mathbf{a^4b^4}$, is what remains factored out on the outside of the parentheses: $\mathbf{a^4b^4(2a + 6b + 9ac)}$

If we multiplied this out, we'd see that we'd get the original polynomial we started with. Done!

Answer: $a^4b^4(2a + 6b + 9ac)$

3. Hm, let's start by pulling out something easy, like 3! Next, we can see that the only variable common to all the terms is z, and the lowest power of z is 3, so we can safely pull out z^3 from each term.

$$\begin{array}{r|l}
3 & 3z^8 - 12y^2z^5 + 18y^3z^4 - 6z^3 \\ \hline
z^3 & z^8 - 4y^2z^5 + 6y^3z^4 - 2z^3 \\ \hline
 & z^5 - 4y^2z^2 + 6y^3z - 2
\end{array}$$

What we're left with, $z^5 - 4y^2z^2 + 6y^3z - 2$, has no factors common to all of its terms, so we know we've factored it completely, and that's what will go in the parentheses. We get: $3z^3(z^5 - 4y^2z^2 + 6y^3z - 2)$. And if we multiplied this out, we'd get what we started with!

Answer: $3z^3(z^5 - 4y^2z^2 + 6y^3z - 2)$

4. Looks pretty wacky! Let's go ahead and factor out more than one factor at once; how about $7g$? Okay, that second layer is looking a little less daunting… kind of. Next let's factor out h^6, which is a factor of all three terms, and then the only common factor left is k, so we'll factor that out. Now, the last layer, $2k - 6g^3h + 7k^3$, has no common factors to all its terms, so that's what goes in the parentheses!

$$\begin{array}{r|l}
7g & 14gh^6k^2 - 42kg^4h^7 + 49h^6k^4g \\ \hline
h^6 & 2h^6k^2 - 6kg^3h^7 + 7h^6k^4 \\ \hline
k & 2k^2 - 6kg^3h + 7k^4 \\ \hline
 & 2k - 6g^3h + 7k^3
\end{array}$$

And the GCF that we pulled out, the product of the stuff along the left side of our birthday cake, is $7gh^6k$, so we get: $\mathbf{7gh^6k(2k - 6g^3h + 7k^3)}$. If we multiplied this out, we'd see that we'd get the original polynomial we started with. Done!

Answer: $\mathbf{7gh^6k(2k - 6g^3h + 7k^3)}$

DTM from p.302-303

2. First thing we'll do is to distribute that negative sign:

$(a^2 - b^2) - (a^2 + 2b^2) = a^2 - b^2 - a^2 - 2b^2$

And now let's write the subtraction as adding negatives, to make things clearer:

$a^2 + (-b^2) + (-a^2) + (-2b^2)$

Now, combining like terms, we get: $a^2 + (-a^2) = \mathbf{0}$

And: $(-b^2) + (-2b^2) = \mathbf{-3b^2}$

So our answer is $0 + (-3b^2)$; in other words: $\mathbf{-3b^2}$

Answer: $\mathbf{-3b^2}$

3. Again we'll start with distributing the subtraction, and we get:

$(g^2h + 4gh) - (6gh - gh^2) \;=\; g^2h + 4gh - 6gh + gh^2$

Writing the subtraction as "adding a negative," we get:

$g^2h + 4gh + (-6gh) + gh^2$

There is only one kind of like term: ones with gh as the variable part. Let's combine those: $4gh + (-6gh) = -2gh$

So our problem has become: $g^2h + (-2gh) + gh^2$. Now, writing it in descending order (we actually have two choices for which term it can start with, so we'll just leave the first term first), and streamlining the negatives into subtraction, we get: $g^2h + gh^2 - 2gh$.

Answer: $\mathbf{g^2h + gh^2 - 2gh}$

4. There is no subtraction to distribute this time, so we can just drop the parentheses, and while we're at it, let's write the subtraction as "adding negatives" and also underline like terms, since there are so many variable types to keep track of!

$\underline{z^2} + 5z + (-2y^3) + (-3) + 2 + \underline{(-z^2)} + 3y^3 + (-5z)$

Combining the single underline terms, we get: $z^2 + (-z^2) = \mathbf{0}$ Nice!

Combining the wavy underline terms, we get: $5z + (-5z) = \mathbf{0}$ Nice again!

Combining the dotted underline terms, we get: $(-2y^3) + 3y^3 = \boldsymbol{y^3}$

Combining the constants, we get: $(-3) + 2 = \mathbf{-1}$

Putting it all together, we get: $0 + 0 + y^3 + (-1) = \boldsymbol{y^3 - 1}$

Done!

Answer: $\boldsymbol{y^3 - 1}$

5. First we'll distribute the subtraction, and we get:

$2x^6 - 7 + 2x^4 - 3x^5 - x^6 + 3x^3 - 2x^5 - 4$

Now we'll write the subtraction as "adding negatives" and also underline like terms!

$\underline{2x^6} + (-7) + 2x^4 + (-3x^5) + \underline{(-x^6)} + 3x^3 + (-2x^5) + (-4)$

Since two of the terms, $2x^4$ and $3x^3$, didn't have any matching variable types in the expression, I didn't underline them – it just would make things a bit messier!

Combining the single underline terms, we get: $2x^6 + (-x^6) = \boldsymbol{x^6}$

Combining the wavy underline terms, we get: $(-3x^5) + (-2x^5) = \boldsymbol{-5x^5}$

Combining the constants, we get: $(-7) + (-4) = \mathbf{-11}$

And putting that all together in descending order, along with the terms that didn't get combined with anything, we get: $x^6 + (-5x^5) + 2x^4 + 3x^3 + (-11)$. And, streamlining the negatives into subtraction, we get our answer!

Answer: $\boldsymbol{x^6 - 5x^5 + 2x^4 + 3x^3 - 11}$