Hot X: Algebra Exposed

Solution Guide for Chapter 22

Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

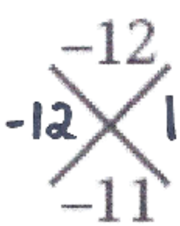
DTM from p.326

2. Let's see, what two numbers' product is 2, and sum is 3? How about 2 and 1?
Yep: $2 \cdot 1 = 2$, and $2 + 1 = 3$. So we write the 2 and 1 on either side of the big X.

Done!

Answer: 2 & 1

3. What are all the factor pairs of 12? Let's see: 1&12, 2&6, and 3&4. We know one must be negative, to give a product of –12, and now which of those might add up to –11? Well, that would be $-12 + 1 = -11$. Great! So we we write 12 and 1 on either side.



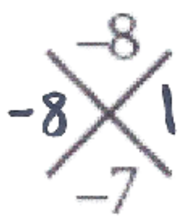
Done!

Answer: –12 & 1

4. We need two numbers that multiply to give –6, and add up to give 1. What are 6’s factor pairs? 6 & 1, and 3 & 2. One of them needs to be negative so that we get a negative product, so how about –3 + 2 = –1. Nope. How about 3 + (–2) = 1. Yep! So we write 3 and –2 on either side. Done!

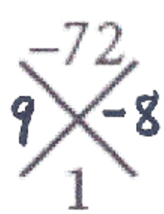
Answer: 3 & –2

5. What factor pairs of –8 are likely to result in a sum of –7? How about –8 and 1? (You’ll see this pattern a lot, where the sum is one number different from the product.) Yep! because –8 + 1 = –7. Done!



Answer: –8 & 1

6. Wow, 72 seems like an awfully big number to have factors that can add up to 1. But what does that mean? Since the product is negative, one factor will be negative and one will be positive, which means we need the factors to be different by one.
Just like how, for example, 333 + (–332) = 1, we can figure out the answer is 9 & –8, since 9 + (–8) = 1. And of course we know from our times tables that 9 times –8 is –72!

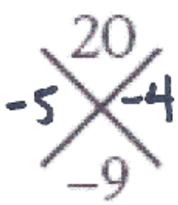


Answer: 9 & –8

7. What two numbers' product is –6 and sum is –1? Our factor pairs are 6 & 1 and 3 & 2. One factor will be negative and one will be positive, to give a negative product. How about –3 & 2? Yep: –3 + 2 = –1.

Answer: –3 & 2

8. The factor pairs of 20 are 20 & 1, 10 & 2, 5 & 4. How can any of these pairs add up to a negative number, –20? How about if they are both negative? Then their product will be positive, but their sum will be negative.
And which of those factor pairs adds up to –9? How about: –5 + (–4) = –9. Yep!



Answer: –5 & –4

9. The factor pairs of 18 are: 18 & 1, 9 & 2, 6 & 3. Again, we'll want both numbers to be negative, so that their product will be positive, but their sum will be negative. And which pair adds up to –9? That's –6 & –3!

Answer: –6 & –3

DTM from p.329

2. So first we'll write our parentheses: (x)(x). What numbers go in the blank spots? We need two numbers that multiply to give 7, but that add up to 8. That would be 7 & 1! And since everything's positive, we'll use + signs inside: **(x + 7)(x + 1)**. Let's check the answer by multiplying it out (use FOIL!) and we get:

$(x + 7)(x + 1) = x^2 + x + 7x + 7 = x^2 + 8x + 7$. Yep!

Answer: (x + 7)(x + 1)

3. First we'll write our parentheses: (w)(w). What numbers go in the blank spots? We need two numbers that multiply to give –12, but that add up to 1, right? (That was an invisible coefficient!) How about the factor pair 4 & 3? Then we could do –3 + 4 = 1. Great! Filling in the –3 & 4, we get: **(w – 3)(w + 4)**. Let's check our answer by multiplying it out, and we get:

$(w - 3)(w + 4) = w^2 + 4w - 3w - 12 = w^2 + w - 12$

Great!

Answer: (w – 3)(w + 4)

4. First we'll write our parentheses: (h)(h). What numbers go in the blank spots? We need two numbers that multiply to give 11, but that add up to –12. How about the factor pair 11 & 1? Then we could do –11 + (–1) = –12 Great!

Filling those in, we get: **(h – 1)(h – 11)**. Let's check our answer by multiplying it out, and we get: $(h - 1)(h - 11) = h^2 - 11h - h + 11 = h^2 - 12h + 11$. Yes!

Answer: (h – 1)(h – 11)

5. First we'll write our parentheses: (x)(x). What numbers go in the blank spots? We need two numbers that multiply to give –15, but that add up to 2, right? How about the factor pair 5 & 3? Then we could do 5 + (–3) = 2.

Filling those in, we get: **$(x - 3)(x + 5)$**. Let's check our answer by multiplying it out, and we get: $(x - 3)(x + 5) = x^2 + 5x - 3x - 15 \;=\; x^2 + 2x - 15$

Answer: $(x - 3)(x + 5)$

6. First we'll write our parentheses: $(y \quad)(y \quad)$. What numbers go in the blank spots? We need two numbers that multiply to give –15, but that add up to –2, right? How about the factor pair 5 & 3? Then we could do $-5 + 3 = -2$.

Filling those in, we get: **$(y + 3)(y - 5)$**. Let's check our answer by multiplying it out, and we get:

$(y + 3)(y - 5) = y^2 - 5y + 3y - 15 \;=\; y^2 - 2y - 15$

Answer: $(y + 3)(y - 5)$

7. Writing our parentheses: $(x \quad)(x \quad)$. What numbers go in the blank spots? We need two numbers that multiply to give 18, but that add up to –11. Well, 18's factor pairs are 18 & 1, 9 & 2, and 6 & 3. How about the factor pair 9 & 2?

Then we could do $-9 + (-2) = -11$.

Filling those in, we get: **$(x - 2)(x - 9)$**. Let's check our answer by multiplying it out, and we get:

$(x - 2)(x - 9) = x^2 - 9x - 2x + 18 \;=\; x^2 - 11x + 18$. Yep!

Answer: $(x - 2)(x - 9)$

8. First we'll write our parentheses: $(g \quad)(g \quad)$. What numbers go in the blank spots? We need two numbers that multiply to give –18 and add up to 7. Hm, looking at 18's factor pairs from the previous problem, how about the factor pair 9 & 2 (we know that one of them needs to be negative). Then we could do $9 + (-2) = 7$.

Filling those in, we get: **$(g - 2)(g + 9)$**. Let's check our answer by multiplying it out, and we get:

$(g - 2)(g + 9) = g^2 - 2g + 9g - 18 \;=\; g^2 + 7g - 18$. Great!

Answer: $(g - 2)(g + 9)$

9. Writing our parentheses, we have: $(x \quad)(x \quad)$. What numbers go in the blank spots? We need two numbers that multiply to give 18, but that add up to –9. We'll want two negative numbers, so that they multiply to be positive but will add up to be negative. Well, 18's factor pairs are 18 & 1, 9 & 2, and 6 & 3. How about the factor pair 6 & 3? Then we could do $-6 + (-3) = -9$.

Filling those in, we get: $\mathbf{(x-3)(x-6)}$. Let's check our answer by multiplying it out, and we get:

$(x-3)(x-6) = x^2 - 6x - 3x + 18 = x^2 - 9x + 18$. Yep!

Answer: $\mathbf{(x-3)(x-6)}$

DTM from p.336

2. So first we'll write our parentheses, and since we know that the only factor pair of 2 is 2 & 1, we can fill in the spots before the x's: $(2x \quad)(x \quad)$. So far so good? Now, we know that the last spots need to multiply to give 1, so we only have one option: 1 and 1! So that's: $(2x \quad 1)(x \quad 1)$. Do we want addition or subtraction in there? Since the middle term, $3x$, is positive AND the last term, 1, is positive, we should have + signs in both spots: $\mathbf{(2x+1)(x+1)}$. Let's check our answer by multiplying it out:

$(2x+1)(x+1) = 2x^2 + 2x + x + 1 = 2x^2 + 3x + 1$. Great!

Answer: $\mathbf{(2x+1)(x+1)}$

3. Since 7's only factor pair is 7 & 1, we can write this for our parentheses:
$(7h \quad)(h \quad)$. How about the numbers in the final slots? We need 2 & 1, since they will multiply to give 2. But which one goes where? Well, we want to somehow end up with $15h$ as the middle term, so let's put the 2 where it will multiply times the 7, so we can get 14! That would be: $(7h \quad 1)(h \quad 2)$. Do we want addition or subtraction in there?

Since the middle term, $15h$, is positive AND the last term, 2, is positive, we should have + signs in both spots: $\mathbf{(7h+1)(h+2)}$. Let's see if this thinking got us the right answer:

$(7h+1)(h+2) = 7h^2 + 14h + h + 2 = 7h^2 + 15h + 2$. Yes!

Answer: $\mathbf{(7h+1)(h+2)}$

4. This time, there are a few choices for the first spots: 6 & 1, or 2 & 3, but there's only one choice for the last spots: 1 & 1. So we have two options for how these parentheses could look: $(6w \quad 1)(w \quad 1)$ or $(3w \quad 1)(2w \quad 1)$. How can we end up with $5w$ in the middle? Since all the original terms are positive, we can only use + signs, so let's go with the $3w$ and $2w$. That would be: $\mathbf{(3w + 1)(2w + 1)}$. Let's see if it worked!

$(3w + 1)(2w + 1) = 6w^2 + 3w + 2w + 1 = 6w^2 + 5w + 1$. Yes, we figured out how to factor it!

Answer: $\mathbf{(3w + 1)(2w + 1)}$

5. Since we know that the only factor pair of 2 is 2 & 1, we can fill in the spots before the x's: $(2x \quad)(x \quad)$. Now, we know that the last spots need to multiply to give –2, so we'll use 2 & 1, but which spots do they go in, and which one will be negative? Well, we want to end up with $-3x$ as a middle term, and here are our options:

$(2x - 2)(x + 1)$

$(2x + 2)(x - 1)$

$(2x - 1)(x + 2)$

$\mathbf{(2x + 1)(x - 2)}$

If you multiply them out, you'll see that only the last one works to give us the correct middle term, $-3x$:

$(2x + 1)(x - 2) = 2x^2 - 4x + x - 2 = 2x^2 - 3x + 1$. And voila!

Answer: $\mathbf{(2x + 1)(x - 2)}$

6. Hm, with 10 and 9, this one has like a gazillion options for the parentheses! 10's factor pairs are 10 & 1 and 5 & 2. And 9's factor pairs are 9 & 1 and 3 & 3. All we really know is: $(\_\_m \qquad)(\_\_m \qquad)$. Not much to start with… but this middle term, $33m$, can give us a clue. What numbers could result in a number like 33? Well, if a 3 multiplies times a 10, then we'd be close! So let's try the pairs 10 & 1 for the 10, and 3 & 3 for the 9. That would be: $(10m \quad 3)(m \quad 3)$. Yep, in this situation, the 10 multiplies times a 3. Since the original polynomial only has positive terms, we have to use + signs in both spots, which would be: $\mathbf{(10m + 3)(m + 3)}$. Does this work? Let's check it out!

$(10m + 3)(m + 3) = 10m^2 + 30m + 3m + 9 \quad = \quad 10m^2 + 33m + 9$

Well, how about that! Sometimes the ones that seem the hardest are actually easier because of the clues they give…

Answer: $(10m + 3)(m + 3)$

7. At first glance, this one seems to have like a gazillion options for both the first and second spots… but wait, we can pull out a factor from all terms! Let's pull out –5, so we get: $-10y^2 + 15y + 10 = -5(2y^2 - 3y - 2)$. Looking better!

Now let's factor just what's inside the parentheses: $y^2 - 3y - 2$. Hey, this is the same as #5, with y instead of x. So we already know that $y^2 - 3y - 2 = (2y + 1)(y - 2)$. So our entire problem has factored like this:

$-10y^2 + 15y + 10 =$ **$-5(2y + 1)(y - 2)$**

Done!

Answer: $-5(2y + 1)(y - 2)$

8. This one only has two options for the second slot: 1 & 1! But for the first slots we could have 4 & 1 or 2 & 2. So our parentheses will look like one of these:

$(4g \quad 1)(g \quad 1)$ or $(2g \quad 1)(2g \quad 1)$

And we know that one of the signs will be positive, and the other will be negative, since the 1's must multiply to give –1 at the end. The middle term we want is $-3g$. How can we get that? How about $-4g + g = -3g$? Then let's use $(4g \quad 1)(g \quad 1)$, and make sure to put the negative sign where it will multiply times the $4g$.

That would be: $(4g + 1)(g - 1)$. Let's test our theory:

$(4g + 1)(g - 1) \quad = \quad 4g^2 - 4g + g - 1 \quad = \quad 4g^2 - 3g - 1$. Fantastic!

Answer: $(4g + 1)(g - 1)$

9. For this one, we have just one choice for the first slot: 3 & 1, and the same for the second slot: 3 & 1. Okay, but which positions do they go in?

So our parentheses will look like either $(3x \quad 1)(x \quad 3)$ or $(3x \quad 3)(x \quad 1)$. But since we want a middle term of $8x$, let's make sure two 3's multiply times each other so we get $9x$, which seems like it could get us that $8x$! That would be: $(3x \quad 1)(x \quad 3)$. Since the final term, –3, is negative, one sign will be positive and the other will be negative. We'll want a positive $8x$, so let's make sure the $9x$ is positive That would be: $(3x - 1)(x + 3)$. Let's test it by multiplying it out: $(3x - 1)(x + 3) = 3x^2 + 9x - x - 3 = 3x^2 + 8x - 3$. Great!

Answer: $(3x - 1)(x + 3)$

DTM from p.341

2. Okay, there's no GCF to pull out of this one. To factor this by grouping, let's note that $a = 12$, $b = -1$, and $c = -6$. So $ac = -72$ and $b = -1$, and now we want to find a pair of numbers whose product is 74 and whose sum is –1.

Hey, since 8 X 9 = 72, how about 8 + (–9) = –1. So then we split up the middle term, $-x$, with these two coefficients.

Our problem becomes: $12x^2 - x - 6 = \mathbf{12x^2 + 8x - 9x - 6}$

Good progress!

Now, let's factor the first two terms by pulling out the GCF, $4x$, and we get:

$4x(3x + 2) - 9x - 6$. Now let's factor the second two terms by pulling out the GCF, –3, and we get: $4x(3x + 2) - 3(3x + 2)$.

And now we see that the GCF of this new "two-termed" expression is $(3x + 2)$, so pulling THAT out, we get:

$\mathbf{4x}(3x + 2) \mathbf{- 3}(3x + 2) = (3x + 2)(\mathbf{4x - 3})$

So, does this work? Let's multiply it out: **$(3x + 2)(4x - 3)$**, and we get:

$(3x + 2)(4x - 3) = 12x^2 - 9x + 8x - 6 = 12x^2 - x - 6$. Great!

Answer: $(3x + 2)(4x - 3)$

[This is equivalent to the answer in the back of the book: $(4x - 3)(3x + 2)$]

3. So in this case, $ac = -60$ and $b = 11$. What pair of numbers has the product –60 and the sum of 11? Hm, let's look at the factor pairs of 60:

60 & 1, 30 & 2, 20 & 3, 15 & 4, 10 & 6.

Hey, how about 15 & 4, since 15 X (–4) = –60 and 15 – 4 = 11. So, splitting up the middle term with those numbers, our problem becomes:

$10x^2 + 11x - 6 \quad = \quad 10x^2 + 15x - 4x - 6$

Time to factor the first two terms by factoring out their GCF, $5x$, and we get:

$5x(2x + 3) - 4x - 6$

Next we pull out the GCF from the second pair of terms, which is -2, and we get:

$5x(2x + 3) - 2(2x + 3)$

And now the GCF of this new expression is $(2x + 3)$, so we pull THAT out, and we get:

$\mathbf{(2x + 3)(5x - 2)}$

Does this work?

Let's multiply it out: $(2x + 3)(5x - 2) \quad = \quad 10x^2 - 4x + 15x - 6 \quad = \quad 10x^2 + 11x - 6$.

Great!

Answer: $\mathbf{(2x + 3)(5x - 2)}$

4. So in this case, $ac = -6$ and $b = -5$. What pair of numbers has the product -6 and the sum of -5? How about -6 and 1? So let's split the middle term with those numbers, and our problem becomes: $3x^2 - 5x - 2 \quad = \quad \mathbf{3x^2 - 6x + 1x - 2}$. (It'll be easier to proceed if we take the hint and leave the coefficient of 1 there.)

Now let's factor out the GCF of the first two terms, $3x$, and we get:

$3x(x - 2) + 1x - 2$

How can we factor the second two terms? Well, the GCF of $1x$ and -2 is just 1, so let's use that! We get: $3x(x - 2) + 1(x - 2)$. Now the GCF of this expression's terms is $(x - 2)$, so factoring THAT out, we get: $(x - 2)(3x + 1)$.

Let's see if it works, by multiplying it out:

$\mathbf{(x - 2)(3x + 1)} \quad = \quad 3x^2 + 1x - 6x - 2 \quad = \quad 3x^2 - 5x - 2$. Yep!

Answer: $\mathbf{(x - 2)(3x + 1)}$

5. In this case, $ac = -120$ and $b = 2$. What pair of numbers has the product -120 and the sum of 2? Well, 120's factor pairs are: 120 & 1, 60 & 2, 40 & 3, 20 & 6, 12 & 10…

Hey, let's stop here, because it sure seems like 12 and 10 will be our winners, since $12 - 10 = 2$. So let's split the middle term with those numbers, and our problem becomes:

$8x^2 + 2x - 15 = \mathbf{8x^2 + 12x - 10x - 15}$

Time to factor the first two terms by pulling out their GCF, $4x$, and we get:

$4x(2x + 3) - 10x - 15$

Next we factor out the second two terms' GCF, which is -5, and we get:

$4x(2x + 3) - 5(2x + 3)$.

Finally, notice that the GCF of this expression is $(2x + 3)$, so let's pull THAT out, and we get: **$(2x + 3)(4x - 5)$**. Did we factor it correctly? Let's check by multiplying it out:

$(2x + 3)(4x - 5) \quad = \quad 8x^2 - 10x + 12x - 15 \quad = \quad 8x^2 + 2x - 15$. Done!

Answer: $(2x + 3)(4x - 5)$