Hot X: Algebra Exposed

Solution Guide for Chapter 3

Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

DTM from p.34-35

2. Let's find the GCF of these two terms. First we'll look at the coefficients; the GCF of 2 and 10 is **2**. Now the only variable factor they have in common is b, and comparing b to b^2, $\boldsymbol{b}$ wins! So the GCF of the two terms is $\mathbf{2}\boldsymbol{b}$, so we'll rewrite our original expression with $2b$ pulled of each one. Let's put parentheses around the whole thing so we can more easily do reverse distribution in a moment, and we get: $2ab + 10b^2 = (\mathbf{2}\boldsymbol{b} \cdot a + \mathbf{2}\boldsymbol{b} \cdot 5b)$ Pulling out the $2b$ with reverse distribution now, we get: $\mathbf{2}\boldsymbol{b}(a + 5b)$. Let's check our work, and multiply it out and make sure we get what we started with:

$$\mathbf{2}\boldsymbol{b}(a + 5b) \;=\; 2ba + 2b(5b) \;=\; 2ab + 10b^2\text{, yep!}$$

Answer: $\mathbf{2}\boldsymbol{b}\mathbf{(}\boldsymbol{a}\mathbf{ + 5}\boldsymbol{b}\mathbf{)}$

3. Since 6 and 7 have no common factors, their GCF is just 1. Next, we'll look at the variable factors. Comparing c^2 and c, $\boldsymbol{c}$ wins! Comparing d and d^2, $\boldsymbol{d}$ wins! So the GCF, and hence the thing we'll pull out, is $1 \cdot c \cdot d = \boldsymbol{cd}$. So rewriting it with this pulled out, and putting parentheses around the whole thing, we get $(\boldsymbol{cd} \cdot 6c - \boldsymbol{cd} \cdot 7d)$. What's left over when we pull $\boldsymbol{cd}$ out? Well, we get $6c - 7d$ inside our parentheses, so our answer should be: $\boldsymbol{cd}(6c - 7d)$. But let's multiply it out just to be sure:

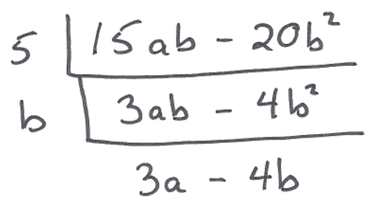
$$cd(6c - 7d) \;=\; cd(6c) - cd(7d) \;=\; 6c^2d - 7cd^2\text{, yep!}$$

Answer: $\boldsymbol{cd}\mathbf{(6}\boldsymbol{c}\mathbf{ - 7}\boldsymbol{d}\mathbf{)}$

4. This is a strange problem because we could just subtract these two terms, right? But let's keep the method the same and see what happens. Since 7 and 2 have no common factors, the coefficients' GCF will be **1**. How about the variables? Well, they both share x, and comparing x to x, $\boldsymbol{x}$ wins! This means the thing we pull out of the party is: $1 \cdot x = \boldsymbol{x}$. and so rewriting it and putting parentheses around the whole thing, we get: $(\boldsymbol{x} \cdot 7 - \boldsymbol{x} \cdot 2)$, so doing reverse distribution, we get: $x(7 - 2)$, and this just equals $x \cdot 5 = \mathbf{5}\boldsymbol{x}$. And how about that? If we had just subtracted $7x - 2x$ to begin with, we'd have also gotten $5x$. Just nice to see how things are connected sometimes. ☺ (See p.35 for more on this idea!)
Answer: **5*x***

DTM from p.37

2. Let's factor this with the birthday cake method: first let's pull out a factor of 5 and write it on the side, so our next layer is $3ab - 4b^2$. The only other factor they share is b, so let's write that on the left and our final layer is $3a - 4b$.



Multiplying the left side gives us the GCF, $5b$, which is what "pulled out of the party," and the bottom layer is what goes in the parentheses, so we get: $\mathbf{5}\boldsymbol{b}\mathbf{(3}\boldsymbol{a}\mathbf{ - 4}\boldsymbol{b}\mathbf{)}$. Let's multiply it out just to be sure: $5b(3a - 4b) = 5b(3a) - 5b(4b) = 15ab - 20b^2$, yep!
Answer: $\mathbf{5}\boldsymbol{b}\mathbf{(3}\boldsymbol{a}\mathbf{ - 4}\boldsymbol{b}\mathbf{)}$

3. For this one, let's start by drawing our first cake layer and factoring out bcd. I mean, why not? The next layer then becomes $12b - 18$. The two terms still share a common factor of 6, so let's factor that one out next, and our final layer becomes $2b - 3$, whose terms are now relatively prime, and that tells us we've factored the expression completely.

$$\begin{array}{r|l} bcd & 12b^2cd - 18bcd \\ \hline 6 & 12b - 18 \\ \hline & 2b - 3 \end{array}$$

The GCF is the left side multiplied out: $6bcd$, and the bottom layer goes in the parentheses, so we get: $\mathbf{6}\boldsymbol{bcd}\mathbf{(2}\boldsymbol{b}\mathbf{ - 3)}$. Let's multiply it out to make sure we get what we started with: $6bcd(2b - 3) = 6bcd(2b) - 6bcd(3) = 12b^2cd - 18bcd$. Yep!

Answer: $\mathbf{6}\boldsymbol{bcd}\mathbf{(2}\boldsymbol{b}\mathbf{ - 3)}$

4. Hm, a monster problem! Ok, let's start simple, and just factor out **6** from each term on the first layer of our "cake." Then we'll see that each term shares at least one b and one d, so let's factor out ***bd*** from the next layer and see what we get. We end up with $2bc - 3c + 4$, which has no more common factors to *all* of its terms, so we know we've found the bottom layer in our "cake."

$$\begin{array}{r|l} 6 & 12b^2cd - 18bcd + 24bd \\ \hline bd & 2b^2cd - 3bcd + 4bd \\ \hline & 2bc - 3c + 4 \end{array}$$

The GCF is the product of the left side of the cake: $6bd$, and the stuff on the bottom ends up in the parentheses, so the answer is: $\mathbf{6}\boldsymbol{bd}\mathbf{(2}\boldsymbol{bc}\mathbf{ - 3}\boldsymbol{c}\mathbf{ + 4)}$. Let's multiply this out with distribution to make sure we get what we started with:

$6bd(2bc - 3c + 4) \;=\; 6bd(2bc) - 6bd(3c) + 6bd(4) \;=\; 12b^2cd - 18bcd + 24bd$. Yep!

Answer: $\mathbf{6}\boldsymbol{bd}\mathbf{(2}\boldsymbol{bc}\mathbf{ - 3}\boldsymbol{c}\mathbf{ + 4)}$

5. Very confusing looking, but I think we can all agree that each term has a factor of "x" in it, right? So let's draw the first layer of our cake and go ahead and pull out just x. Next, we can see that each term has a factor of "y" in it, so let's pull that out, and see where we

stand. We get $xy + x + y - 1$, and since there is no common factor to all of its terms, this is the last layer, and hence, this is what will go in our parentheses.

$$\begin{array}{r|l} x & x^2y^2 + x^2y + xy^2 - xy \\ \hline y & xy^2 + xy + y^2 - y \\ \hline & xy + x + y - 1 \end{array}$$

The GCF is the product of the stuff on the left side, so our answer is: $\boldsymbol{xy(xy + x + y - 1)}$.

Just to make sure we got the right answer, let's multiply it out:

$xy(xy + x + y - 1) \;=\; xy(xy) + xy(x) + xy(y) - xy(1) \;=\; x^2y^2 + x^2y + xy^2 - xy$. Yep!

Answer: $\boldsymbol{xy(xy + x + y - 1)}$