Hot X: Algebra Exposed

Solution Guide for Chapter 8

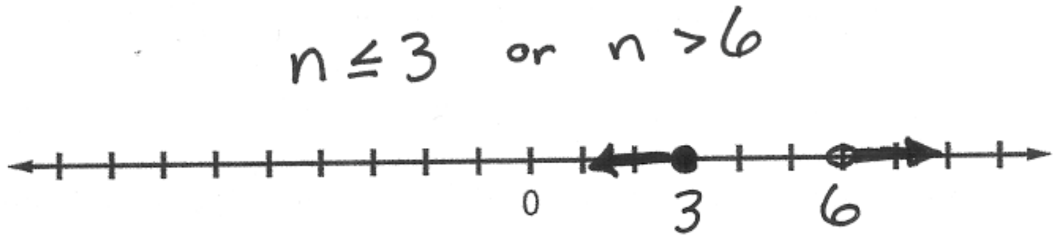
Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

DTM from p.105

2. Since we see the word "or", we know it's a disjunction, which means the entire expression will be satisfied if either condition (inequality) is satisfied… it's pretty easygoing like that. (Think of Daisy from earlier in the chapter!) So we graph all values that satisfy *either* condition.

For the first inequality, $n \leq 3$, we see the $\leq$ which means we'll be filling in the dot at the number 3, and then we will extend the ray to the left, because we want to include all values smaller than 3. For the second part of the disjunction, $n > 6$, the $>$ tells us that we'll use an open dot, and then include all values larger than 6. And that's it!

Answer: It's a disjunction



3. The only thing that has changed is that we see "and" instead of "or." And that's a big change! This is a conjunction, and that means a value must satisfy BOTH conditions

(Remember Connie from earlier this chapter) in order to be a solution. But how could a number be both smaller than or equal to 3, and ALSO larger than 6? It's impossible! So there's no solution, and nothing to graph.

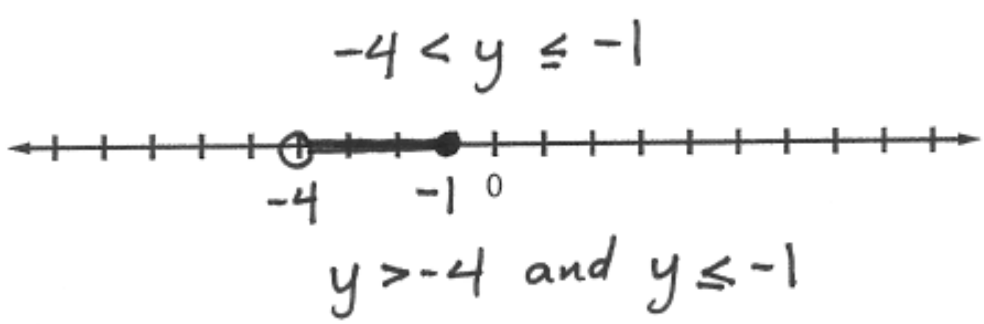
Answer: no solution

4. Ah, we're in synchronized swimming form, which is only possible with conjunctions. In order to solve and graph this solution (find the values of y that will make this a true statement), we need to solve for y, by doing things to all three "sides" of this inequality. To isolate y, let's first subtract 3 from all three segments, and we'll get:
$-8 < 2y \leq -2$. Then, dividing everyone by 2, we get: $\mathbf{-4 < y \leq -1}$.

Just because it's good to understand, notice that $-4 < y \leq -1$ is the same statement as "$y > -4$ <u>and</u> $y \leq -1$." (And no, the direction of that first inequality isn't a typo! Look at the placement of the y and the -4!)

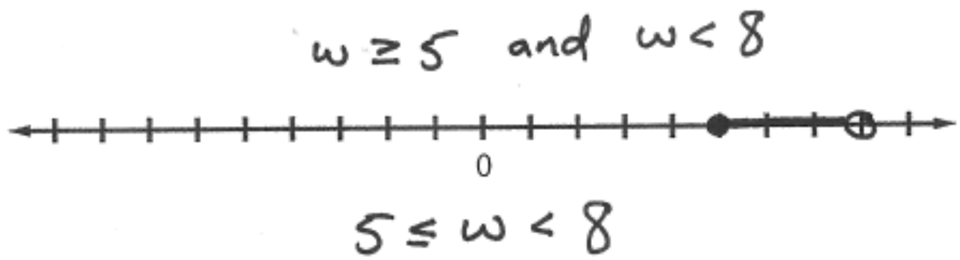
Now that y is isolated, we're ready to graph, and we need values in between -4 and -1, including -1 (closed dot because of $\leq$) but not including -4 (open dot because of $<$).

Answer: It's a conjunction



5. In order to isolate w, let's add 7 to all "sides" and we get: $5 \le w < 8$. And notice that this is the same statement as "$w \ge 5$ and $w < 8$." So we know it's a conjunction! And notice that we'll use a closed dot at 5 and an open dot at 8.

Answer: It's a conjunction



DTM from p.111

2. So the first step with these problems is to find the boundary points, which we can do by changing the inequality to an equals sign, and solving that: $|y - 1| = 2$. Using our "inside guts" strategy from the last chapter, we know this will be a true statement if either of these two mini equations are true.

Equation #1: $y - 1 = 2 \rightarrow y = 3$

Equation #2: $y - 1 = -2 \rightarrow y = -1$

So our boundary points are $\boldsymbol{y = -1, 3}$. Because the original inequality is $\ge$, we'll use closed dots at those two numbers on the graph. So, we can graph those two boundary points on the number line, and then in order to solve and graph the original inequality, we need to find out if the original inequality is represented by the values between the two points, or the values on either side. To do that, we can test a point in the original inequality and see if it results in a true statement!

Let's pick a nice point in the middle of the two points, like 0. Plugging it into the original inequality, we get:

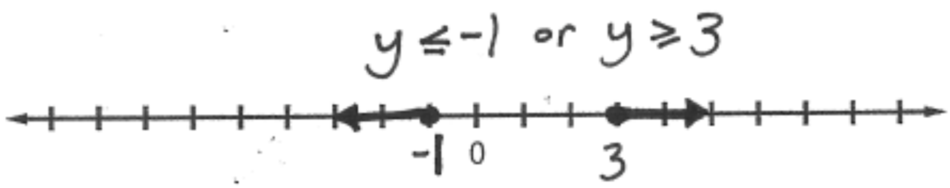
$|\boldsymbol{y} - 1| \ge 2 \quad \rightarrow \quad |\mathbf{0} - 1| \ge 2\ ? \quad \rightarrow \quad |-1| \ge 2\ ? \quad \rightarrow \quad 1 \ge 2?$ Nope!

That means that none of the values in between the boundary points will satisfy the original inequality, and that the points on the outside will satisfy it. We can test a point on the outside, just to double check. Let's pick 5:

$|\boldsymbol{y} - 1| \geq 2 \rightarrow |\mathbf{5} - 1| \geq 2$? $\rightarrow |4| \geq 2$? $\rightarrow 4 \geq 2$? Yep!

So that means that any of values on the outside of the boundary points, $y \leq -1$ or $y \geq 3$, will indeed satisfy the original inequality. It will look like two rays extending in either direction, which is a disjunction.

Answer: It's a disjunction: $y \leq -1$ or $y \geq 3$



3. So, first we'll find the boundary points by writing "=" instead of "<" and solving the equation $|a - 3| = 2$. And this is true when the inside guts are equal to 2 or –2:

Equation #1: $a - 3 = 2 \rightarrow a = 5$

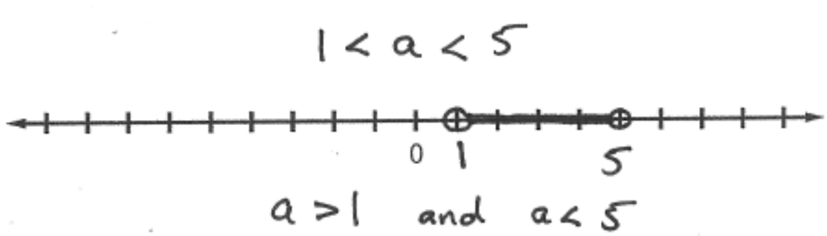
Equation #2: $a - 3 = -2 \rightarrow a = 1$

So our boundary points are $\boldsymbol{a} = \mathbf{1, 5}$. And since the inequality is <, we'll use open dots at those points. Let's test the region in between with a point like, hmm I don't know, how about 4:

$|\boldsymbol{a} - 3| < 2 \rightarrow |\mathbf{4} - 3| < 2$? $\rightarrow |1| < 2$? $\rightarrow 1 < 2$? Yep!

So the region between the boundary points is the solution, which means it's a conjunction. Done!

Answer: It's a conjunction: $1 < \boldsymbol{a} < 5$



4. Looks pretty strange! Let's remain calm and do this one step at a time. Remember we first need to isolate the bars, which they aren't yet, because of the negative sign on the outside! Now, if that negative sign were on the inside of the bars, we wouldn't do anything about it; it would be part of the "inside guts." But as it is, we need to deal with the negative sign in order to isolate the bars, so we'll multiply both sides by –1. And since we've got an inequality, that means we'll be reversing the direction of the inequality symbol! (Remember the Mirror rule from p.100?) So we get:

$$-|x| \geq -1 \rightarrow |x| \leq 1$$

With me so far? Okay, now we're ready to get our boundary points, which means solving this, but with an equals sign: $|x| = 1$, and we hardly have to set up our two equations to solve this one. It's true when $x = -1$ or 1!

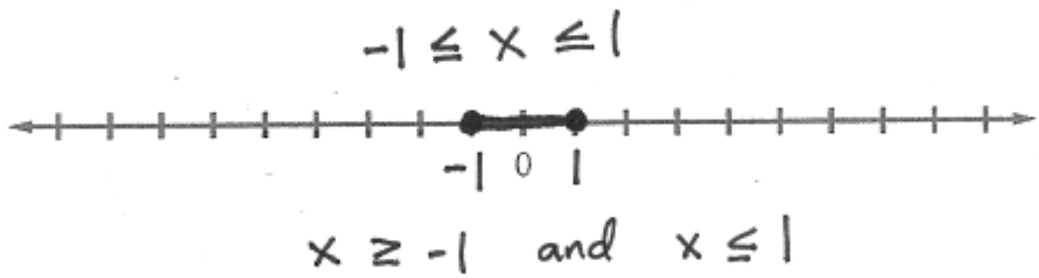
So our boundary points are $\boldsymbol{x = -1, 1}$. And we'll be using closed dots because of the original inequality's symbol.

Testing the region in between the two points (let's use $x = 0$) in the original inequality, we get:

$-|\boldsymbol{x}| \geq -1 \rightarrow -|\mathbf{0}| \geq -1$? $\rightarrow -0 \geq -1? \rightarrow 0 > -1?$ Yep!

So the region in between the boundary points is our solution, and this means it's a conjunction.

Answer: It's a conjunction: $\boldsymbol{-1 \leq x \leq 1}$



5. First we must isolate the bars, by adding 1 to both sides, and then dividing both sides by 2, and we get:

$2|2n + 1| - 1 > 5 \rightarrow 2|2n + 1| > 6 \rightarrow \underline{|2n + 1| > 3}$

Great, now we're ready to find boundary points, which we'll do by solving the equation $|2n + 1| = 3$. Looking at the inside guts, we get:

Equation #1: $2n + 1 = 3 \rightarrow 2n = 2 \rightarrow n = 1$

Equation #2: $2n + 1 = -3 \rightarrow 2n = -4 \rightarrow n = -2$

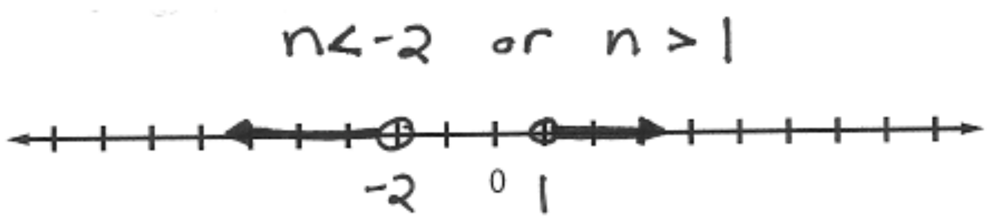
So our boundary points are $\boldsymbol{n = -2, 1}$. And we'll use open dots because of the ">".

Let's test the region in between the points, using $n = 0$. And because it's the *identical statement* as the original inequality (it's just simplified), we can use the equation that is underlined above, to do the testing:

$|2\boldsymbol{n} + 1| > 3 \rightarrow |2\mathbf{(0)} + 1| > 3$? $\rightarrow |0 + 1| > 3$? $\rightarrow |1| > 3$? $\rightarrow 1 > 3$? Nope!

So the region on the *outsides o*f the boundary points, –2 & 1, is our solution: Any of the values smaller than –2 or larger than 1 will satisfy our original inequality. And that's a disjunction!

Answer: It's a disjunction: $\boldsymbol{n < -2}$ or $\boldsymbol{n > 1}$



5. First we have to isolate the bars. First we'll subtract 1 from both sides and then we'll divide both sides by –4, which means (because of the mirror rule) the direction of the inequality symbol will reverse!

We get: $-4|h - 3| + 1 > 13 \rightarrow -4|h - 3| > 12 \rightarrow \underline{|h - 3| < -3}$

Now that the bars are isolated, we can find our boundary points. But wait! We can't continue because it's impossible for an absolute value expression to be less than a negative number. That underlined inequality has no solution. You'll see in Step 1 on p.107 that we must pay attention to this sort of thing before we move on – otherwise we could get a totally wrong answer!

Answer: No solution