"Hot X: Algebra Exposed"

Supplemental PDF: <u>Scientific Notation</u>

...as promised on p. 270

So, on pp. 268-270 in *Hot X: Algebra Exposed*, we defined "scientific notation" and talked about how to write really big and really small numbers with it. Let's talk about it in more detail now!

Recall that when we raise 10 to a positive power, that just tells us how many zeros it has after the "1." Also, if we raise 10 to a negative power, the exponent tell us how many places we move the decimal point to the left from the "1."

Positive Powers of 10	Negative Powers of 10
$10^1 = \mathbf{10}$	$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = \mathbf{0.1}$
$10^2 = \mathbf{100}$	$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = \mathbf{0.01}$
$10^3 = \mathbf{1000}$	$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = \mathbf{0.001}$
$10^4 = \mathbf{10{,}000}$	$10^{-4} = \frac{1}{10^4} = \frac{1}{10{,}000} = \mathbf{0.0001}$

And as we saw on p.269, to write a number in scientific notation, we just take the decimal point and move it enough places until the number is between 1 and 10 (but not equal to 10), and then however many places we moved it, that number becomes the exponent on 10!

Let's practice:

Doing the Math #1

Write these in scientific notation. I'll do the first one for you.

1. 0.0000000205

<u>Working out the solution:</u> We need to move the decimal until we get a number between 1 and 10. So if we move the decimal **<u>8</u>** places to the right, we'll get: 2.05, which is indeed between 1 and 10. But will we use 10^8 or 10^{-8}? Our original number is very small, so we should end up with a negative exponent. So, 0.0000000205 = $\mathbf{2.05 \times 10^{-8}}$**.**

Answer: $\mathbf{2.45 \times 10^{-8}}$

2. 790,000,000,000

3. 12,300

4. 0.9

5. 0.000000000801

(Answers to DTM #1 at bottom)

Now let's go in reverse, and we'll write scientific notation numbers as normal, "expanded form" numbers. This time, the exponent will tell us how many places we need to move the decimal point in order to drop the "power of 10" and write these numbers without scientific notation.

QUICK NOTE:

When converting numbers from scientific notation into expanded form, move the decimal place until there are no more numbers, and then *continue counting* by adding zeros. When you're done counting, write the decimal point down. And notice if it's a big or small number – that will help you remember which way to move the decimal point.

Doing the Math #2

Write these in expanded notation. I'll do the first one for you.

1. 3.01×10^{-9}

Working out the solution: Since the exponent is negative, I know we should end up with a very small number, so we'll move the decimal to the left – in fact, to the left 9 places. Counting one place over, we get .301, so continuing with 8 more, we'll end up adding 8 zeros to the left of that leading 3, and we'll get: 0.00000000301.

Answer: 0.00000000301

2. 2.3×10^{5}

3. 3.4×10^{-5}

4. 5.06×10^{7}

5. 7.08×10^{-7}

(Answers to DTM #2 at the bottom)

Operations with Scientific Notation: Adding/Subtracting

In science class, you may be asked to add, subtract, multiply or divide these crazy-looking numbers. For example:

$$2.3 \times 10^{-9} + 4.5 \times 10^{-9}$$

Are we even allowed to do that? How about this problem instead: $2.3x + 4.5x$. You'd know it was $5.8x$, right? After all, these are like terms, so we just add their coefficients.[1] Well, x could have any value; in fact, it could even equal 10^{-9}. So that means:

$$2.3 \times 10^{-9} + 4.5 \times 10^{-9} = \mathbf{5.8 \times 10^{-9}}$$

So, if the notation ever confuses you, just think about variables – we can always add like terms!

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[1] To review how to combine like terms, check out chapter 9 in *Kiss My Math*.

Step by Step

Adding/Subtracting Numbers in Scientific Notation

Step 1. Make sure that 10 has the same power on both numbers; this means they are *like terms*.

Step 2. Thinking of the powers of 10 as the variables, now we just add/subtract the "coefficients," and keep the same power of 10 for the answer.

Step 3. Make sure the answer is still in scientific notation. You may have to adjust the decimal and power, so that the "coefficient" is between 1 and 10. Done!

QUICK NOTE

If you want to add/subtract two numbers in scientific notation, and their powers of 10 are different, then try taking one of the numbers out of scientific notation for a moment, in order to make the powers the same. I'll show you what I mean.

Step by Step in Action

Subtract $1.7\times10^{-5} - 8\times10^{-6}$

Step 1. We have to make both powers of 10 the same. Let's rewrite 8×10^{-6} so that it has –5 as the exponent instead of –6. If we're going to make the exponent *bigger* (since –5 >

–6), essentially multiplying the entire number by 10, then we'll have to compensate equally by making the coefficient smaller by moving its decimal place to the left, essentially dividing the entire number by 10, so we're back where we started.

So: $8 \times 10^{-6} = 0.8 \times 10^{-5}$. Now our problem looks like:

$$1.7 \times 10^{-5} - 0.8 \times 10^{-5}$$

Step 2. Now that the powers of 10 are equal, it's as if we had *like terms*, so we can subtract "coefficients": 1.7 – 0.8 = 0.9. And keeping the power of 10 the same, we get:

$$0.9 \times 10^{-5}$$

Step 3. But this isn't exactly in scientific notation, since 0.9 is not between 1 and 10. So we'll need to move the decimal place to the right, so it becomes 9 (multiplying the entire number by 10). This means we'll need to compensate by decreasing the exponent by one, so it becomes –6 (dividing the entire number by 10). $0.9 \times 10^{-5} = 9 \times 10^{-6}$

Answer: $\mathbf{9 \times 10^{-6}}$

This stuff can be tricky, but remember, all we just did was to first multiply by 10 and then divide by 10, so that we didn't change the *value* of our answer; just its form!

QUICK NOTE

Notice that in step 1, if only we'd changed the first number to 17×10^{-6}, instead of changing the second number, we would have ended up with an answer in scientific notation and wouldn't have had to do step 3. But I wanted to show you all the different ways it can go!

WATCH OUT

It's so easy to make a mistake in step 1, like when we converted 8×10^{-6} to 0.8×10^{-5} in the above example. Go slowly, and always think to yourself something like: "Hm, 0.8 is *smaller* by a factor of 10, and 10^{-5} is *bigger* by a factor of 10, so we haven't changed the value." And you'll be fine!

Doing the Math #3

Add/subtract the following and leave your answer in scientific notation. (Don't even think about writing these puppies in expanded form!) I'll do the first one for you.

1. $6 \times 10^{12} + 1.5 \times 10^{14}$

Working out the solution: Looks like we're going to have to change one of their exponents, since they don't match. Let's change the first term. Since increasing the exponent from 12 to 14 essentially *multiplies* the entire thing by 100, we'll have to also *divide* the entire thing by 100, by moving the decimal point two places to the left. This way we don't change the value of the expression. So: $6 \times 10^{12} = 0.06 \times 10^{14}$. Now our problem looks like: $0.06\times10^{14} + 1.5\times10^{14} = \mathbf{1.56\times10^{14}}$. Since 1.56 is between 1 and 10, we're done!

Answer: $\mathbf{1.56\times10^{14}}$

2. $2.3\times10^{59} + 9\times10^{59}$

3. $8.1\times10^{-19} + 7.1\times10^{-19}$

4. $1.1\times10^{29} - 9\times10^{29}$

5. $1.1\times10^{-29} - 9\times10^{-30}$

(Answers to DTM #3 at bottom)

Multiplying/Dividing with Scientific Notation

So how do we do these kinds of problems?

$$(4\times10^{5})(2\times10^{-4}) = ? \quad \text{or} \quad \frac{6\times10^{17}}{2\times10^{10}} = ?$$

Let's just rewrite these, to make them seem a bit more palatable:

$$4\cdot10^{5}\cdot2\cdot10^{-4} = ? \quad \text{or} \quad \frac{6}{2}\times\frac{10^{17}}{10^{10}} = ?$$

Ah, yes, better. For $4\cdot10^{5}\cdot2\cdot10^{-4}$, we can rearrange the factors to get this: $4\cdot2\cdot10^{5}\cdot10^{-4} = 8\cdot10^{5}\cdot10^{-4}$. Then, since we have similar bases being multiplied, we just *add* their exponents: 5 + (–4) = 1, and we get: $8 \cdot 10^5 \cdot 10^{-4} = 8 \cdot 10^{5+(-4)} = 8 \cdot 10^1$,

which we can write as 80, or in scientific notation: 8×10^1.

If you want to write all that out each time, you are welcome to: It's a great way to "see" what's going on. But now you can see that to multiply scientific notation numbers, we can just multiply the "coefficients" together and separately multiply the 10's by adding their exponents, and voila, we're done!

The same quotient rule for exponents applies to dividing scientific notation numbers. We just handle the front numbers separately from the powers of 10. We can subtract the exponents, using the quotient rule from p.252 of *Hot X: Algebra Exposed*, or just think of it as canceling powers of 10:

$$\frac{6}{2} \times \frac{10^{17}}{10^{10}} = 3 \times 10^{17-10} = 3 \times 10^7$$

Just like with multiplying, we can deal with the front numbers "separately" from the powers of 10, and then stick 'em back together afterwards. Nice!

Doing the Math #4

Simplify these and leave your answer in scientific notation. I'll do the first one for you.

1. $(4.9 \times 10^{-8}) \div (0.7 \times 10^{10})$

Working out the solution: Let's write this division in fraction form: $\frac{4.9 \times 10^{-8}}{0.7 \times 10^{-10}}$ and then split it up: $\frac{4.9}{0.7} \times \frac{10^{-8}}{10^{-10}}$. For the first fraction, why don't we get rid of the decimals by multiplying top and bottom by 10 (which doesn't change the fraction's value), and we'll then apply the quotient rule and subtract the exponents. So:

$\frac{4.9}{0.7} \times \frac{10^{-8}}{10^{-10}} \;=\; \frac{49}{7} \times 10^{-8-(-10)} \;=\; \mathbf{7 \times 10^{2}}$. We know this is really just 700, but the problem is asking for scientific notation, after all.

Answer: $\mathbf{7 \times 10^{2}}$

2. $(9 \times 10^{-7})(3 \times 10^{2})$

3. $(9 \times 10^{-7}) \div (3 \times 10^{2})$

4. $\frac{2.4 \times 10^{-16}}{0.3 \times 10^{8}}$

(Answers to DTM #4 at bottom)

Takeaway Tips

-When adding/subtracting numbers in scientific notation, so long as the powers of 10 are the same, just add/subtract the front numbers – the "coefficients' – just as with like terms.

-When multiplying/dividing numbers in scientific notation, you can deal with the front numbers separately from the powers of 10, and then squish everything back together at the end.

Answer Key:

DTM #1

2. 7.9×10^{11}
3. 1.23×10^{4}
4. 9×10^{-1}
5. 8.01×10^{10}

DTM #2

2. 230,000
3. 0.000034
4. 50,600,000
5. 0.000000708

DTM #3

2. 11.3×10^{59}
3. 15.2×10^{-19}
4. -7.9×10^{29}
5. 2×10^{-30} (See the example on p.6 above!)

DTM #4

2. 2.7×10^{-4}

3. 3×10^{-9}
4. 8×10^{-24}