"Hot X: Algebra Exposed"

Supplemental PDF: <u>Percents/Investments</u>

...as promised on p.215

It's pretty crazy how we can increase money by 10% and then decrease it by 10% and end up with less money that we started with. We saw this sort of thing happen all the time in Chapter 15 of *Hot X: Algebra Exposed*, but that doesn't make it any less wacky!

Let's look at the algebra behind the money, to find out how that money goes away, and just how much goes. We'll do this by using only variables. And since things are going to look pretty hairy, let's use a little box "□" for one of the variables, it'll just make it easier to handle.

So, instead of 10% (in other words, 0.10), if "r" is the rate of increase or decrease, then if we start with some amount of money, let's say □, and <u>increase</u> it by r percent, then the amount of money we have *after* the increase is $\square + \square r$, right? (See p. 209 for a review of the formula $P + Pr$.)

Well, now we've got more money than we started with. So next, let's start over with our current amount of money, this big messy $[\square + \square r]$, and let's say we <u>decrease</u> this new, big, messy amount of money by "r" percent. Seems like we should end up with what we started with: □. But watch and see what happens! Remember, when we <u>decrease</u> an amount by r, we use the formula $P – Pr$.

So, plugging this new messy thing in for P in the formula $P – Pr$, we get:

$$P \quad - \quad Pr$$

$$\rightarrow \; [\square + \square r] - [\square + \square r]r$$

Let's simplify this, by first distributing that outside *r* and then the negative sign:

$$\square + \square r - [\square r + \square r^2]$$

$$= \square + \square r - \square r - \square r^2$$

$$\mathbf{= \square - \square \boldsymbol{r}^2}$$

Wow, so after first increasing $\square$ by *r* percent, and then decreasing it by *r* percent, we end up with the same amount we started with, minus r^2 percent of what we started with!

Try doing this yourself, but start by first decreasing an amount by an interest rate of *r*, and then increasing it by an interest rate of *r*. You see, *it doesn't matter* if we first increased by *r*, and then decreased by *r*, or if we do it in reverse: You end up with the same amount of money either way.

Think about what that means for a second: When we start out with $\square$ dollars, then increase by *r* and then decrease by *r* (or vice versa), we end up losing $\square r^2$ dollars along the way. Does that check out? If we start with $50, increase by 10% and then decrease by 10%, then $\square = 50$, and $r = 0.1$, then we'd end up losing $50(0.1)^2 = 50(0.01) = 50$ cents.

So we'd end up with $49.50 – and yep, that's the correct answer for #2 from p.214 of *Hot X: Algebra Exposed*!

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EXTRA BONUS: If we *lose* money when the interest rate is the same for a decrease and then an increase, then how can we break even? Is there a formula for how much you'd have to increase by, in order to make up for a certain percent decrease?

Yep! And I didn't find this in any finance or math book – I just derived it from the formula we already know – which you're about to do, too! No matter how much money □ we invest, if it decreases by a percent, "*r*", then in order to end up □ dollars again, our investment must increase by *what* percent? Let's call this new percent "*R*." We know it must be bigger than *r*, but how much bigger?

If we decrease □ by *r* percent, we're left with: $\square - \square r$ dollars, right? Now, let's take this messy amount and we'll say that the new amount increases by *R* percent. What would the final amount of money look like? Well, $\square - \square r$ is our new "*P*," and we'd get $P + PR$, in other words: $[\square - \square r] + [\square - \square r]R$. Now let's see what *R* would have to be, in order for this entire thing to be equal to our original amount of money, □. By setting that whole thing equal to □, we can solve for the rate *R* that would make this a true statement – and we can find the rate that will give us our money back!

$$[\square - \square \boldsymbol{r}] + [\square - \square \boldsymbol{r}]\boldsymbol{R} = \square$$

Now solve for *R*, in terms of the other variables. After distributing the *R*, you'll see that you can divide both sides of the equation by □, (or multiply both sides by $\frac{1}{\square}$ and distribute), and the □'s disappear completely!

$$\square - \square r + \square R - \square rR = \square$$

$$\rightarrow\ 1 - r + R - rR = 1$$

Next we isolate *R*, by getting all the stuff *not* involving *R* to the other side:

$$\rightarrow\ R - rR = r + 1 - 1$$

Now we factor out R, as we learned to do in Chapter 3 of *Hot X*: *Algebra Exposed.* Pull out of that party, girl!

$$\rightarrow R(1 - r) = r$$

$$\rightarrow \boldsymbol{R = \frac{r}{1 - r}}$$

So what does this mean? It means that if our money first decreases by r percent, then it would have to increase by $\frac{r}{1-r}$ percent, in order to break even. And it doesn't matter what the actual amount of money was! This formula will work *every time* (As long as ◻ doesn't equal zero, since we had to divide by that amount. But it would be pretty silly to start with zero dollars and try to increase it or decrease it by a percent – it would always stay at zero!)

So for instance, if we started with \$50 and decreased it by 20% (so, $\boldsymbol{r = 0.2}$), we'd get:

$$\$50 - \$50(20\%) \quad = \quad 50 - 50(0.2) \quad = \quad 50 - 10 \quad = \quad \mathbf{\$40}$$

And in order to get back to \$50, we now know that we'd need to increase \$40 by more than 20%, in fact, we'd have to increase the money by $\frac{r}{1-r}$ percent, which, since $r = 0.2$ in this case, equals:

$$\frac{r}{1-r} = \frac{0.2}{1-0.2} = \frac{0.2}{0.8} = \frac{2}{8} = \frac{1}{4} = 0.25 = \mathbf{25\%}$$

And does that work? Sure does!

Since 25% of \$40 is \$10, we'd end up with \$40 + \$10 = **\$50**.

Now, is there any need to memorize these formulas we've figured out? No way! Like I said, I've never even seen this last one printed anywhere. But after seeing this stuff and working with it, you'll automatically "get" the regular problems faster, and you may even be asked to do examples that will seem much more intuitive now. Like, you'll have a context from which to say, "Yeah, that seems like it's the right answer," which is a great feeling. To go exploring and discovering in math is a great thing, because it leads to a deeper understanding and a stronger sharpening of the mind.

Ms. Exponent would be so proud.[1]

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[1] For more on Ms. Exponent, see Chapters 16-17 in *Kiss My Math*. She'll visit again in Chapters 17-18 of *Hot X*, too!