"Hot X: Algebra Exposed"

Supplemental PDF: Happy Unit Cancellation

…as promised on p. 207

So, units are really awesome. I admit, I used to think of them as sort of "secondary" to the numbers in a problem. I mean, is the answer "8" or "8 bananas"? Who cares, as long as we get 8, right?

Well… units are more helpful than you'd think. If you haven't yet already, check out the "What's the Deal" on p.206-207 of *Hot X* to remember what unit multipliers are (or for a full review, check out Chapter 19 of *Math Doesn't Suck*) and to how units cancelling explain the formulas we use for "rate of work" and $r \cdot t = d$ motion word problems, and even p.231 of *Hot X* to see how units cancelling help us understand how to set up our equations in mixture/liquid word problems.

But wait, there's more! Yes, happy unit cancellation can explain even more things that we may have been wondering about. For example, on p.200 of *Hot X*, the Quick Note tells us that these two expressions are equivalent to each other:

$$\frac{5 \text{ jobs}}{4 \text{ hours}} = \frac{\frac{5}{4} \text{ jobs}}{1 \text{ hour}}$$

…and we move seamlessly back and forth between these types of expressions throughout Chapter 14 in *Hot X*. But how do we know these expressions mean the same thing? Do we really have to take it on faith? If you were pressed, could you say *why* they are the same?

Hm, well, from our work with complex fractions (check out Chapter 9 in *Math Doesn't Suck* for a review), you already know that $\frac{\frac{5}{4}}{1} = \frac{5}{4}$. All we'd have to do is to rewrite the first fraction as: $\frac{\frac{5}{4}}{\frac{1}{1}}$, apply the "means and extremes" method on it, and we'd get $\frac{5}{4}$.

Even still, it might seem strange to you that we can simply leave the units where they are, and rewrite the fractions as we're allowed to with normal fraction rules. It always seemed strange to me, anyway. In case you're interested, I'm going to show you how to get from one to the other, using a little happy unit cancellation. And what are our *units* here? Not bananas this time, but *jobs* and *hours*!

So, if we know that a gal can do $\frac{5}{4}$ of a job in 1 hour, and we were asked how many jobs she could do in 4 hours, since we know from the big equation on p.199 that rate $\cdot$ time = total jobs done, we can multiply the *rate* times the *time*, and watch the hours units cancel, leaving us with the "jobs" answer we were looking for:

$$\frac{\frac{5}{4}\ jobs}{1\ hour} \bullet 4\,hours \quad = \quad \frac{\frac{5}{4}\ jobs}{1\ hour} \bullet \frac{4\,hours}{1} \quad = \quad \frac{\frac{5}{4}\ jobs \cdot 4}{1 \cdot 1} \quad = \quad \mathbf{5\ jobs}$$

And now since we have learned that she can do 5 jobs in 4 hours, we could also write her rate as: $\frac{5\ jobs}{4\ hours}$.

Ta-da! Hence, $\dfrac{\frac{5}{4}\ jobs}{1\ hour}$ is the same rate as $\dfrac{5\ jobs}{4\ hours}$.

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And by the way, if we were asked how many jobs the same gal could do in 10 hours, we could just fill in the new number, and again watch the "hours" units cancel:

$$\frac{5\ jobs}{4\ hours} \bullet 10\ hours \;=\; \mathbf{\frac{50}{4}}\ \textbf{jobs}$$

or

$$\frac{\frac{5}{4}\ jobs}{1\ hour} \bullet 10\,hours \;=\; \frac{\frac{5}{4}\ jobs \cdot 10}{1} \;=\; \mathbf{\frac{50}{4}}\ \textbf{jobs}$$

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More Happy Unit Cancellation

When converting between minutes and seconds, remember how we can use either $\dfrac{1\ min}{60\ sec}$ or $\dfrac{60\ sec}{1\ min}$, depending on what we want to cancel? (See Chapter 19 in *Math Doesn't Suck* to review this.) Well, we can do the same thing with our rate of work fractions; we can use them not just as rates, but we can flip them upside-down for other purposes! For example, if someone wanted to know *how long* it would take this gal to do, say, 15 jobs, we could use the "unit multiplier" $\dfrac{4\ hours}{5\ jobs}$ (instead of $\dfrac{5\ jobs}{4\ hours}$), and watch the "jobs" units cancel this time, leaving us with the "hours" answer we wanted!

$$\frac{4\ hours}{5\ jobs} \bullet 15\ \ jobs = \frac{4 \cdot 15}{5}\ hours\ \ = \mathbf{12\ hours}$$

Of course, we also could have just used the rate $\cdot$ time = total jobs formula, and solved for *t*. Since, as you know, rate of work is *always* expressed at “jobs per time,” we’d get:

$$\frac{5\ jobs}{4\ hours} \cdot t = 15 \text{ jobs}$$

And then we’d solve for *t* (and we’d get 12 hours), much like we did in #1 on p.204-205 in *Hot X*. But look at what’s really going on; in order to isolate t, we shouldn’t just multiply both sides by 4 and divide both sides by 5. What we’re really doing is multiplying both sides by 4 hours and dividing both sides by 5 jobs. In other words, multiplying both sides by $\frac{4\ hours}{5\ jobs}$:

$$\frac{4\ hours}{5\ jobs}\left(\frac{5\ jobs}{4\ hours} \cdot t\right) = \frac{4\ hours}{5\ jobs}(15\ jobs)$$

And then we can see how, not only do the 4’s and 5’s on the left side cancel, but ALL of the units on both sides, except for the final “hours” on the right side, will cancel, too! That’s how we know we’re doing it right –the <u>unit</u> that the problem was asking for is leftover at the end:

$$t = \frac{60}{5}\ hours\ \ = \mathbf{12\ hours}$$

When the units cancel, we know we’re doing something right! It’s amazing how helpful these happy unit cancellations really are. Not all word problems are easy to categorize, and it’s nice to be able to “lean” on the units, who can show us the way!

Thanks for reading. You’re awesome.